

1.6

Angle Pair Relationships

What you should learn

GOAL 1 Identify vertical angles and linear pairs.

GOAL 2 Identify complementary and supplementary angles.

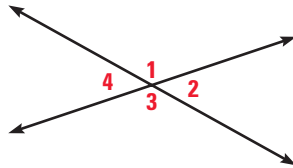
Why you should learn it

▼ To solve **real-life** problems, such as finding the measures of angles formed by the cables of a bridge in Ex. 53.

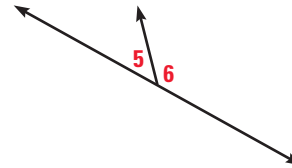
**GOAL 1** VERTICAL ANGLES AND LINEAR PAIRS

In Lesson 1.4, you learned that two angles are *adjacent* if they share a common vertex and side but have no common interior points. In this lesson, you will study other relationships between pairs of angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays. Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays.



$\angle 1$ and $\angle 3$ are vertical angles.
 $\angle 2$ and $\angle 4$ are vertical angles.

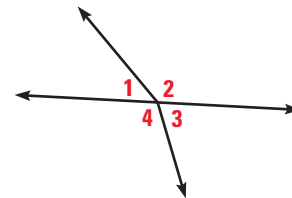


$\angle 5$ and $\angle 6$ are a linear pair.

In this book, you can assume from a diagram that two adjacent angles form a linear pair if the noncommon sides appear to lie on the same line.

EXAMPLE 1 Identifying Vertical Angles and Linear Pairs

- Are $\angle 2$ and $\angle 3$ a linear pair?
- Are $\angle 3$ and $\angle 4$ a linear pair?
- Are $\angle 1$ and $\angle 3$ vertical angles?
- Are $\angle 2$ and $\angle 4$ vertical angles?

**SOLUTION**

- No. The angles are adjacent but their noncommon sides are not opposite rays.
- Yes. The angles are adjacent and their noncommon sides are opposite rays.
- No. The sides of the angles do not form two pairs of opposite rays.
- No. The sides of the angles do not form two pairs of opposite rays.

.....

In Activity 1.6 on page 43, you may have discovered two results:

- *Vertical angles are congruent.*
- *The sum of the measures of angles that form a linear pair is 180° .*

Both of these results will be stated formally in Chapter 2.



EXAMPLE 2 Finding Angle Measures

In the stair railing shown at the right, $\angle 6$ has a measure of 130° . Find the measures of the other three angles.

SOLUTION

$\angle 6$ and $\angle 7$ are a linear pair. So, the sum of their measures is 180° .

$$m\angle 6 + m\angle 7 = 180^\circ$$

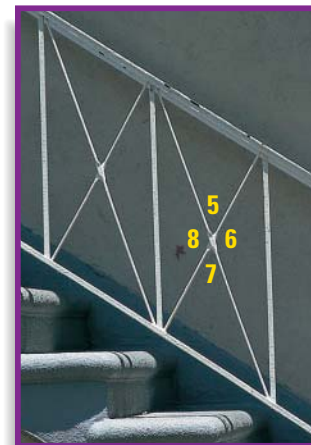
$$130^\circ + m\angle 7 = 180^\circ$$

$$m\angle 7 = 50^\circ$$

$\angle 6$ and $\angle 5$ are also a linear pair. So, it follows that $m\angle 5 = 50^\circ$.

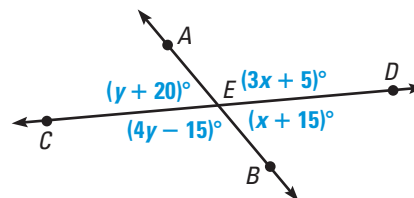
$\angle 6$ and $\angle 8$ are vertical angles. So, they are congruent and have the same measure.

$$m\angle 8 = m\angle 6 = 130^\circ$$



EXAMPLE 3 Finding Angle Measures

Solve for x and y .
Then find the angle measures.



SOLUTION

Use the fact that the sum of the measures of angles that form a linear pair is 180° .

$$m\angle AED + m\angle DEB = 180^\circ$$

$$m\angle AEC + m\angle CEB = 180^\circ$$

$$(3x + 5)^\circ + (x + 15)^\circ = 180^\circ$$

$$(y + 20)^\circ + (4y - 15)^\circ = 180^\circ$$

$$4x + 20 = 180$$

$$5y + 5 = 180$$

$$4x = 160$$

$$5y = 175$$

$$x = 40$$

$$y = 35$$

Use substitution to find the angle measures.

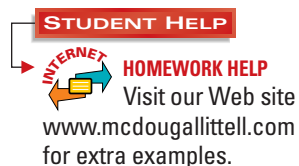
$$m\angle AED = (3x + 5)^\circ = (3 \cdot 40 + 5)^\circ = 125^\circ$$

$$m\angle DEB = (x + 15)^\circ = (40 + 15)^\circ = 55^\circ$$

$$m\angle AEC = (y + 20)^\circ = (35 + 20)^\circ = 55^\circ$$

$$m\angle CEB = (4y - 15)^\circ = (4 \cdot 35 - 15)^\circ = 125^\circ$$

So, the angle measures are 125° , 55° , 55° , and 125° . Because the vertical angles are congruent, the result is reasonable.



GOAL 2 COMPLEMENTARY AND SUPPLEMENTARY ANGLES

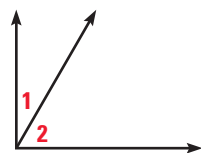
STUDENT HELP

Study Tip

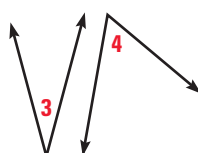
In mathematics, the word *complement* is related to the phrase *to complete*. When you draw the complement of an angle, you are “completing” a right angle. (The word *compliment* is different. It means something said in praise.)

Two angles are **complementary angles** if the sum of their measures is 90° . Each angle is the **complement** of the other. Complementary angles can be adjacent or nonadjacent.

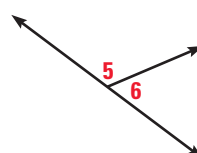
Two angles are **supplementary angles** if the sum of their measures is 180° . Each angle is the **supplement** of the other. Supplementary angles can be adjacent or nonadjacent.



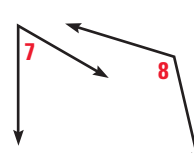
complementary
adjacent



complementary
nonadjacent



supplementary
adjacent



supplementary
nonadjacent

EXAMPLE 4 Identifying Angles

State whether the two angles are complementary, supplementary, or neither.

SOLUTION

The angle showing 4:00 has a measure of 120° and the angle showing 10:00 has a measure of 60° . Because the sum of these two measures is 180° , the angles are supplementary.



EXAMPLE 5 Finding Measures of Complements and Supplements

- Given that $\angle A$ is a complement of $\angle C$ and $m\angle A = 47^\circ$, find $m\angle C$.
- Given that $\angle P$ is a supplement of $\angle R$ and $m\angle R = 36^\circ$, find $m\angle P$.

SOLUTION

- $m\angle C = 90^\circ - m\angle A = 90^\circ - 47^\circ = 43^\circ$
- $m\angle P = 180^\circ - m\angle R = 180^\circ - 36^\circ = 144^\circ$



EXAMPLE 6 Finding the Measure of a Complement

$\angle W$ and $\angle Z$ are complementary. The measure of $\angle Z$ is five times the measure of $\angle W$. Find $m\angle W$.

SOLUTION

Because the angles are complementary, $m\angle W + m\angle Z = 90^\circ$. But $m\angle Z = 5(m\angle W)$, so $m\angle W + 5(m\angle W) = 90^\circ$. Because $6(m\angle W) = 90^\circ$, you know that $m\angle W = 15^\circ$.

GUIDED PRACTICE

Vocabulary Check ✓

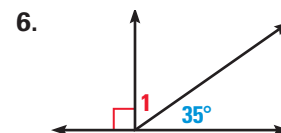
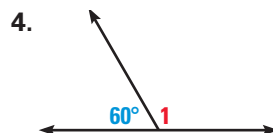
1. Explain the difference between *complementary angles* and *supplementary angles*.


Concept Check ✓

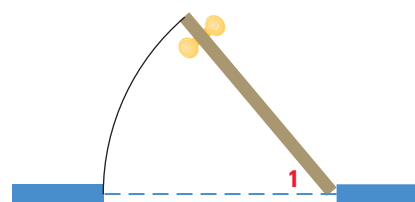
2. Sketch examples of acute vertical angles and obtuse vertical angles.
3. Sketch examples of adjacent congruent complementary angles and adjacent congruent supplementary angles.

Skill Check ✓

FINDING ANGLE MEASURES Find the measure of $\angle 1$.



7.  **OPENING A DOOR** The figure shows a doorway viewed from above. If you open the door so that the measure of $\angle 1$ is 50° , how many more degrees would you have to open the door so that the angle between the wall and the door is 90° ?



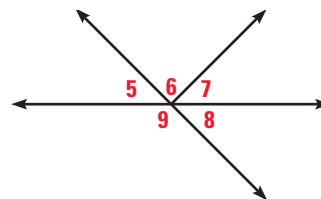
PRACTICE AND APPLICATIONS

STUDENT HELP

➔ **Extra Practice**
to help you master
skills is on p. 804.

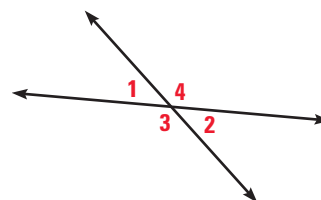
IDENTIFYING ANGLE PAIRS Use the figure at the right.

8. Are $\angle 5$ and $\angle 6$ a linear pair?
9. Are $\angle 5$ and $\angle 9$ a linear pair?
10. Are $\angle 5$ and $\angle 8$ a linear pair?
11. Are $\angle 5$ and $\angle 8$ vertical angles?
12. Are $\angle 5$ and $\angle 7$ vertical angles?
13. Are $\angle 9$ and $\angle 6$ vertical angles?



EVALUATING STATEMENTS Decide whether the statement is *always*, *sometimes*, or *never* true.

14. If $m\angle 1 = 40^\circ$, then $m\angle 2 = 140^\circ$.
15. If $m\angle 4 = 130^\circ$, then $m\angle 2 = 50^\circ$.
16. $\angle 1$ and $\angle 4$ are congruent.
17. $m\angle 2 + m\angle 3 = m\angle 1 + m\angle 4$
18. $\angle 2 \cong \angle 1$
19. $m\angle 2 = 90^\circ - m\angle 3$



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 8–13
Example 2: Exs. 14–27
Example 3: Exs. 28–36
Example 4: Exs. 37–40
Example 5: Exs. 41, 42
Example 6: Exs. 43, 44

FINDING ANGLE MEASURES Use the figure at the right.

20. If $m\angle 6 = 72^\circ$, then $m\angle 7 = \underline{\hspace{1cm}}$.

21. If $m\angle 8 = 80^\circ$, then $m\angle 6 = \underline{\hspace{1cm}}$.

22. If $m\angle 9 = 110^\circ$, then $m\angle 8 = \underline{\hspace{1cm}}$.

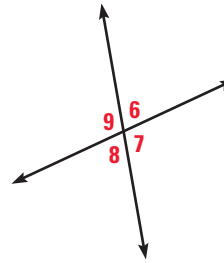
23. If $m\angle 9 = 123^\circ$, then $m\angle 7 = \underline{\hspace{1cm}}$.

24. If $m\angle 7 = 142^\circ$, then $m\angle 8 = \underline{\hspace{1cm}}$.

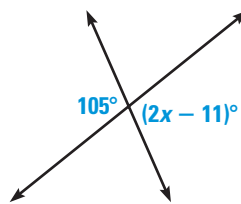
25. If $m\angle 6 = 13^\circ$, then $m\angle 9 = \underline{\hspace{1cm}}$.

26. If $m\angle 9 = 170^\circ$, then $m\angle 6 = \underline{\hspace{1cm}}$.

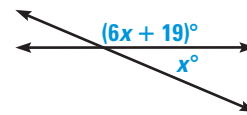
27. If $m\angle 8 = 26^\circ$, then $m\angle 7 = \underline{\hspace{1cm}}$.

**USING ALGEBRA** Find the value(s) of the variable(s).

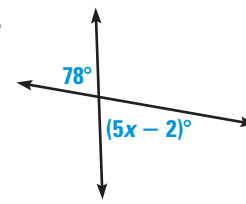
28.



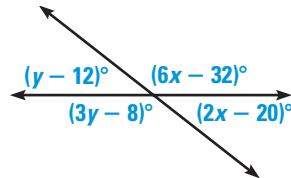
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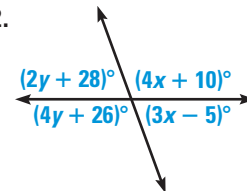
30.



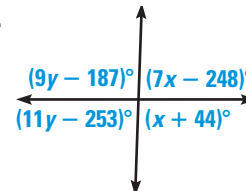
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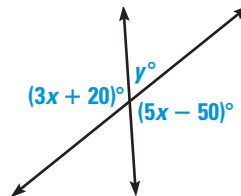
32.



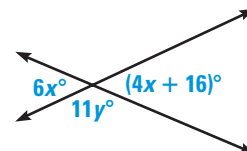
33.



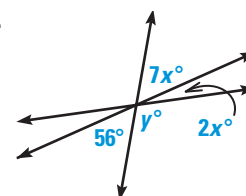
34.



35.



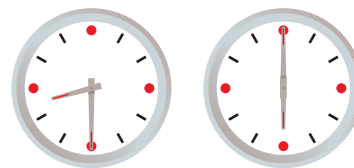
36.

**IDENTIFYING ANGLES** State whether the two angles shown are *complementary*, *supplementary*, or *neither*.

37.



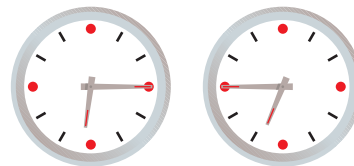
38.



39.



40.





- 41. FINDING COMPLEMENTS** In the table, assume that $\angle 1$ and $\angle 2$ are complementary. Copy and complete the table.

$m\angle 1$	2°	10°	25°	33°	40°	49°	55°	62°	76°	86°
$m\angle 2$?	?	?	?	?	?	?	?	?	?

- 42. FINDING SUPPLEMENTS** In the table, assume that $\angle 1$ and $\angle 2$ are supplementary. Copy and complete the table.

$m\angle 1$	4°	16°	48°	72°	90°	99°	120°	152°	169°	178°
$m\angle 2$?	?	?	?	?	?	?	?	?	?

- 43.  USING ALGEBRA** $\angle A$ and $\angle B$ are complementary. The measure of $\angle B$ is three times the measure of $\angle A$. Find $m\angle A$ and $m\angle B$.
- 44.  USING ALGEBRA** $\angle C$ and $\angle D$ are supplementary. The measure of $\angle D$ is eight times the measure of $\angle C$. Find $m\angle C$ and $m\angle D$.

FINDING ANGLES $\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.

- 45.** $m\angle A = 5x + 8$
 $m\angle B = x + 4$
- 46.** $m\angle A = 3x - 7$
 $m\angle B = 11x - 1$
- 47.** $m\angle A = 8x - 7$
 $m\angle B = x - 11$
- 48.** $m\angle A = \frac{3}{4}x - 13$
 $m\angle B = 3x - 17$

FINDING ANGLES $\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.

- 49.** $m\angle A = 3x$
 $m\angle B = x + 8$
- 50.** $m\angle A = 6x - 1$
 $m\angle B = 5x - 17$
- 51.** $m\angle A = 12x + 1$
 $m\angle B = x + 10$
- 52.** $m\angle A = \frac{3}{8}x + 50$
 $m\angle B = x + 31$


FOCUS ON PEOPLE

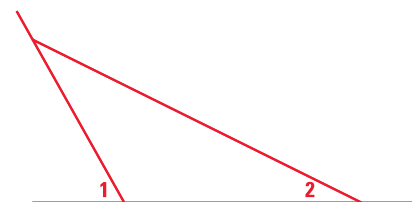



SANTIAGO CALATRAVA, a Spanish born architect, has developed designs for bridges, train stations, stadiums, and art museums.



APPLICATION LINK
www.mcdougallittell.com

- 53.  BRIDGES** The Alamillo Bridge in Seville, Spain, was designed by Santiago Calatrava. In the bridge, $m\angle 1 = 58^\circ$ and $m\angle 2 = 24^\circ$. Find the supplements of both $\angle 1$ and $\angle 2$.



- 54.  BASEBALL** The foul lines of a baseball field intersect at home plate to form a right angle. Suppose you hit a baseball whose path forms an angle of 34° with the third base foul line. What is the angle between the first base foul line and the path of the baseball?

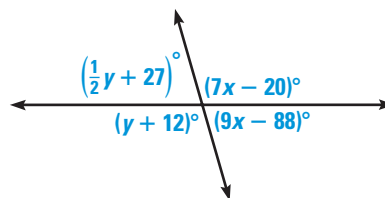
Test Preparation

55. **PLANTING TREES** To support a young tree, you attach wires from the trunk to the ground. The obtuse angle the wire makes with the ground is supplementary to the acute angle the wire makes, and it is three times as large. Find the measures of the angles.

56. **Writing** Give an example of an angle that *does not* have a complement. In general, what is true about an angle that has a complement?

57. **MULTIPLE CHOICE** In the diagram shown at the right, what are the values of x and y ?

- (A) $x = 74, y = 106$
 (B) $x = 16, y = 88$
 (C) $x = 74, y = 16$
 (D) $x = 18, y = 118$
 (E) $x = 18, y = 94$

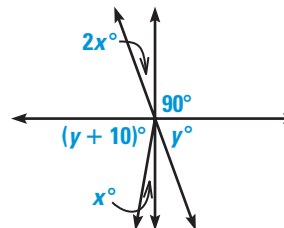


58. **MULTIPLE CHOICE** $\angle F$ and $\angle G$ are supplementary. The measure of $\angle G$ is six and one half times the measure of $\angle F$. What is $m\angle F$?

- (A) 20° (B) 24° (C) 24.5° (D) 26.5° (E) 156°

Challenge

59. **xy USING ALGEBRA** Find the values of x and y in the diagram shown at the right.



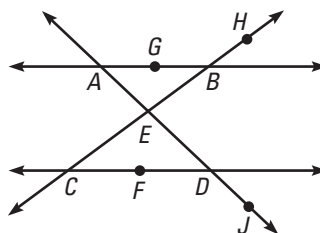
MIXED REVIEW

SOLVING EQUATIONS Solve the equation. (Skills Review, p. 802, for 1.7)

60. $3x = 96$ 61. $\frac{1}{2} \cdot 5 \cdot h = 20$ 62. $\frac{1}{2} \cdot b \cdot 6 = 15$
 63. $s^2 = 200$ 64. $2 \cdot 3.14 \cdot r = 40$ 65. $3.14 \cdot r^2 = 314$

FINDING COLLINEAR POINTS Use the diagram to find a third point that is collinear with the given points. (Review 1.2)

66. A and J
 67. D and F
 68. H and E
 69. B and G



FINDING THE MIDPOINT Find the coordinates of the midpoint of a segment with the given endpoints. (Review 1.5)

70. A(0, 0), B(-6, -4) 71. F(2, 5), G(-10, 7) 72. K(8, -6), L(-2, -2)
 73. M(-14, -9), N(0, 11) 74. P(-1.5, 4), Q(5, -9) 75. S(-2.4, 5), T(7.6, 9)